Valuation of start-up companies in an incomplete market framework

Operations Research 2007
OR in the Service Industry

Saarbrücken, September 6th, 2007

Peter Reichling **Thomas Spengler** **Bodo Vogt**

Otto-von-Guericke-University Magdeburg

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de

www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

Agenda

1. **Motivation**
2. **Rigid and flexible planning in a complete market framework**
3. **Valuation in an incomplete financial market**
4. **Summary**

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de

www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

1 Motivation

- ➔ The financial situation of start-up companies usually allows only equity financing.
- ➔ Firm valuation methods include flexible planning techniques and real option approaches.
- ➔ Corresponding methods assume a complete market or risk neutral decision makers.
- ➔ Due to action choices of the management cash flow streams of start-ups often cannot be duplicated.
- ➔ In general, an exact valuation of such companies is not possible.
- ➔ In this situation our model allows to determine bounds of the firm's value based on market values.

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

Agenda

1. **Motivation**
2. **Rigid and flexible planning in a complete market framework**
3. **Valuation in an incomplete financial market**
4. **Summary**

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

2 Rigid and flexible planning

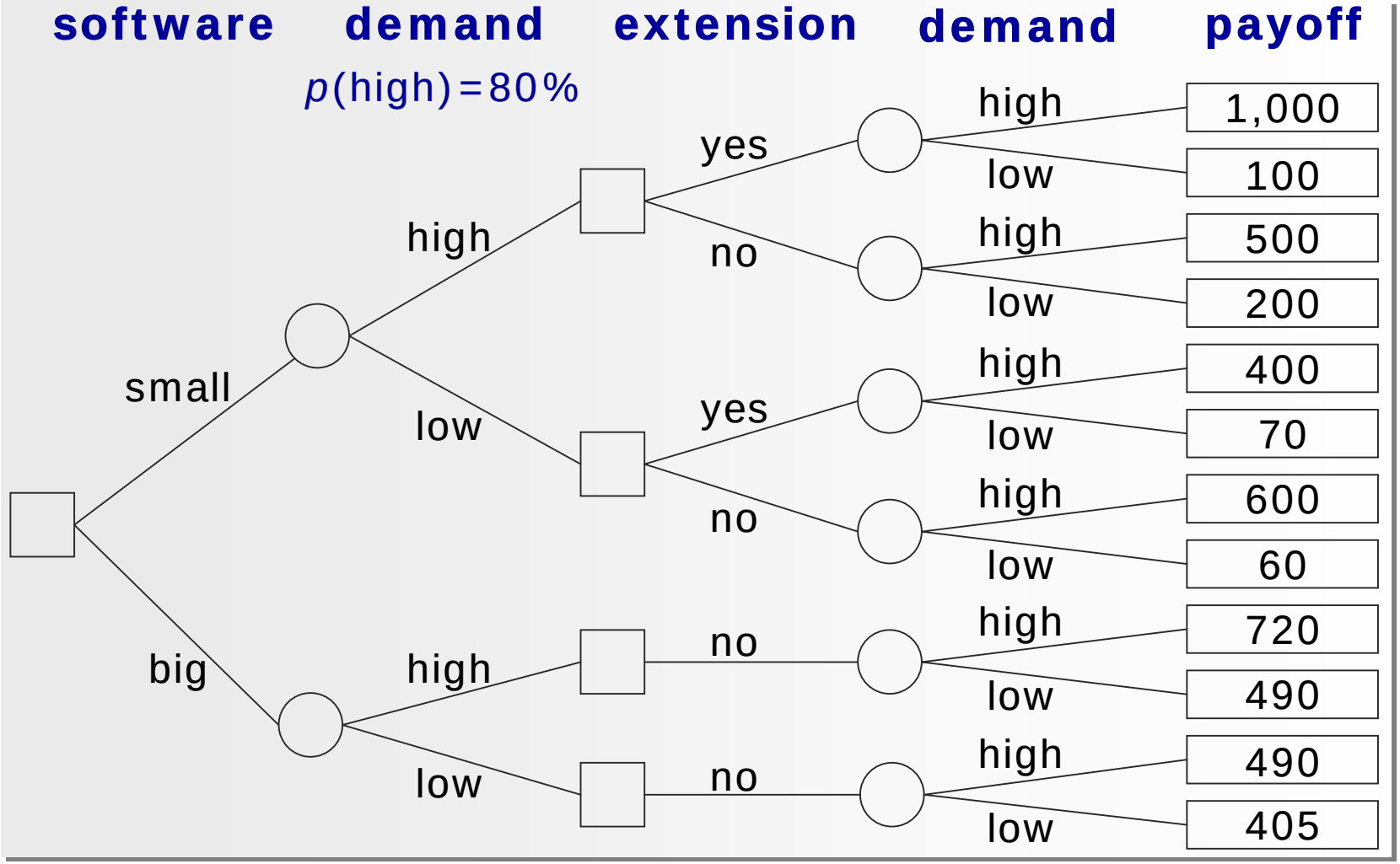
➔ With **rigid planning,** already in the starting point in time the decision maker fixes all decisions he has to make in the complete planning period (independent of future events).

➔ By the use of **flexible planning** techniques only the decision for the 1st period is fixed taking optional plans for subsequent periods into consideration.

➔ Example 2 periods
2 states of nature in each period
5 alternatives

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

Decision tree



Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

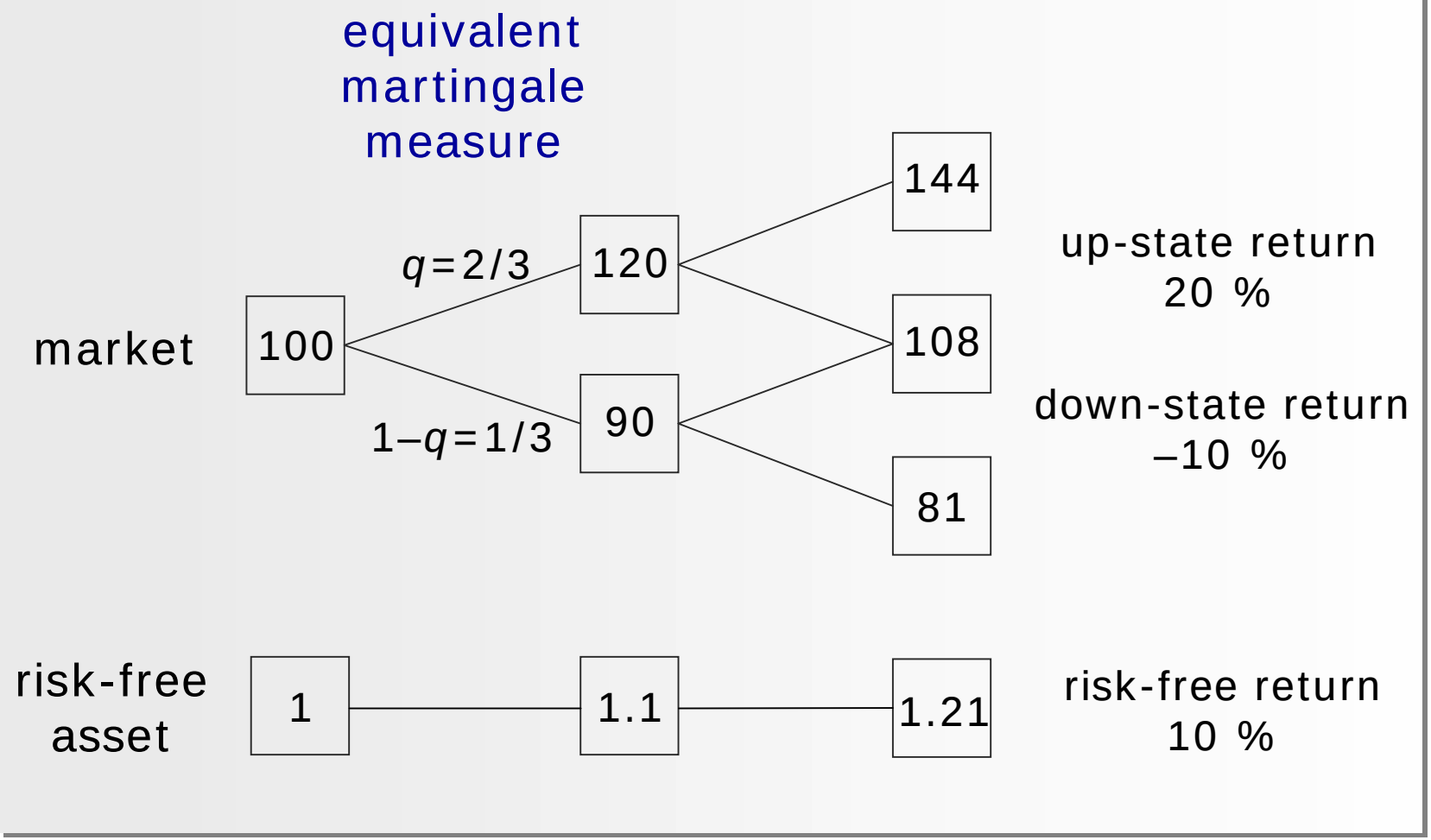
Decision matrix with true probabilities

demand	H_1 H_2	H_1 L_2	L_1 H_2	L_1 L_2	expected value	rank	rank (rigid planing)
prob *p*	64 %	16 %	16 %	4 %			
strategy	**payoff**						
A_1	1,000	100	400	70	722.80	2	1
A_2	500	200	600	60	450.40	4	3
A_3	720	490	490	405	633.80	3	2
A_4	1,000	100	600	60	754.40	1	
A_5	400	70	500	200	355.20	5	

$H/L_{1/2}$	high/low demand in period 1/2
$A_{1/2}$	small software in $t=0$ with/without extension in $t=1$
A_3	big software in $t=0$
$A_{4/5}$	small software in $t=0$ with/without extension in $t=1$ if demand in period 1 is high

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

Binomial market model



Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

Decision matrix under market-oriented valuation

prob	H_1 H_2	H_1 L_2	L_1 H_2	L_1 L_2	expected value	present value	rank	rank (rigid planing)
q	44.4%	22.2%	22.2%	11.1%				
	payoff							
A_1	1,000	100	400	70	563.33	465.56	**3**	**2**
A_2	500	200	600	60	406.67	336.09	4	3
A_3	720	490	490	405	582.78	481.63	**2**	**1**
A_4	1,000	100	600	60	606.67	501.38	1	
A_5	400	70	500	200	326.67	269.97	5	

Duplication of A_4

$t=0$		8.48 market shares – 381.82 zero bonds
$t=1$	(H_1)	6.39 market shares – 250.00 zero bonds
	(L_1)	3.15 market shares – 150.00 zero bonds

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

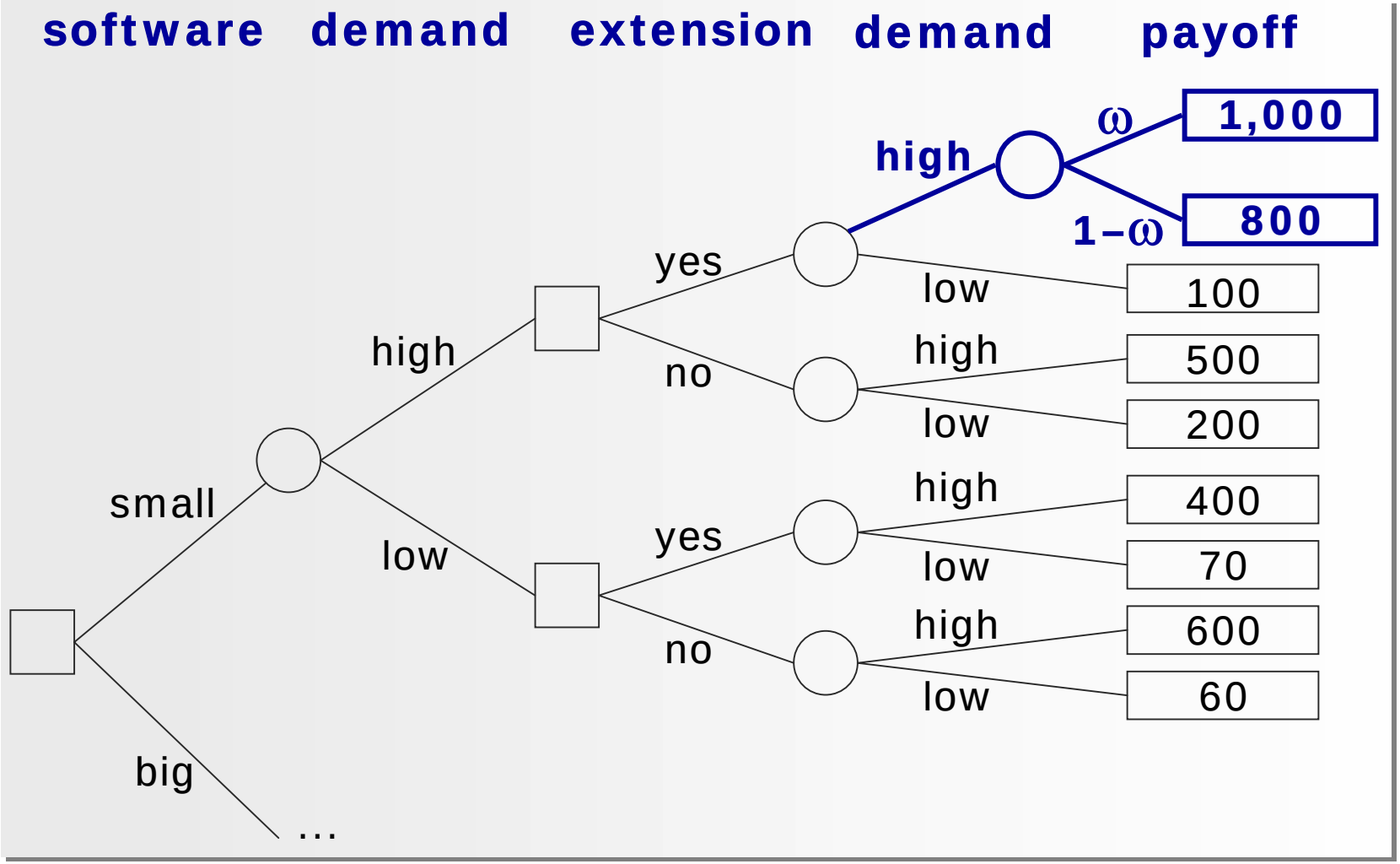
Agenda

1. **Motivation**
2. **Rigid and flexible planning in a complete market framework**
3. **Valuation in an incomplete financial market**
4. **Summary**

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de

3 Valuation in an incomplete market

Suppose a chance move that only affects
the situation of the start-up company, not the market



Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

3 Valuation in an incomplete market

➔ For some final states, payoffs are binary lotteries $[\omega, max, (1-\omega), min]$ with $max > min$ instead of secure payoffs, i.e. only an ambiguous market value exists.

➔ Possible solutions are based on the market values $V(min)$ and $V(max)$:

⇨ Preference based approaches

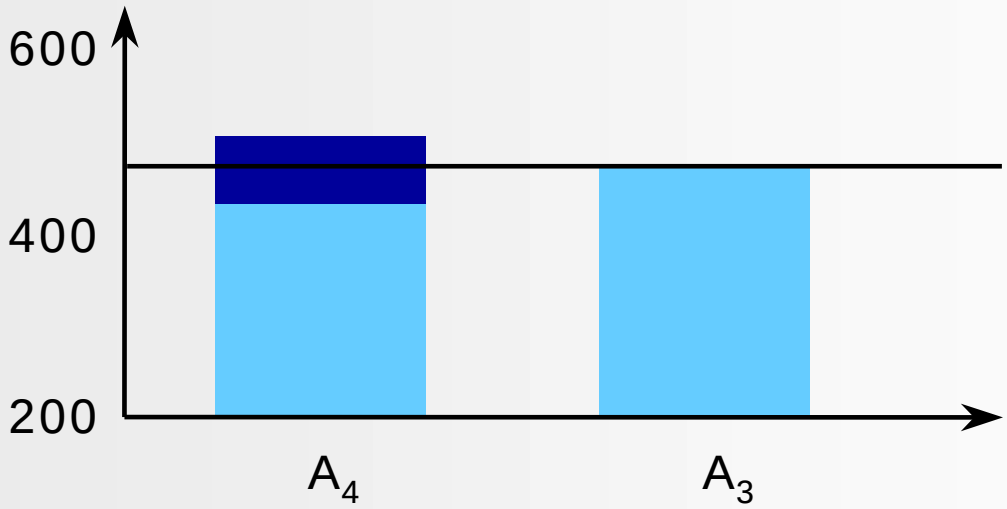
- Determine $E(u([\omega, V(max), (1-\omega), V(min)]))$
- Linear utility function $\omega \cdot V(max) + (1-\omega) \cdot V(min)$
- Hurwicz criterion $\alpha \cdot V(max) + (1-\alpha) \cdot V(min)$

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

3 Valuation in an incomplete market

➔ We only change the payoff for the small software strategy with extension in the states H_1 and H_2 from 1,000 to the binary lottery $[\omega;1{,}000;(1-\omega);800]$.

Present value intervall in an incomplete market



➔ In the case of the Hurwicz criterion a decision maker with $\alpha=0.73$ is indifferent between the alternatives.

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

Agenda

1. **Motivation**
2. **Rigid and flexible planning in a complete market framework**
3. **Valuation in an incomplete financial market**
4. **Summary**

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1

4 Summary

- ➔ The financial situation of start-up companies usually allows only equity financing.
- ⇨ **Firm valuation is needed**
- ➔ Firm valuation methods include flexible planning techniques and real option approaches.
- ⇨ **Market-oriented valuation**
- ➔ Due to action choices of the management cash flow streams of start-ups often cannot be duplicated.
- ⇨ **Ambiguous values**
- ➔ In this situation our model allows to determine bounds of the firm's value.
- ⇨ **Bounds can be computed**
- ⇨ **Apply, e.g. Hurwicz criterion**

Peter.Reichling@ovgu.de
Thomas.Spengler@ovgu.de
Bodo.Vogt@ovgu.de
www.uni-magdeburg.de/finance
www.ww.uni-magdeburg.de/bwl5
www.ww.uni-magdeburg.de/pbwl1